MATHEMATICS MODEL PAPER

SRI A.S.N.M. GOVERNMENT COLLEGE(A), PALAKOL. W.G.DT

(Affiliated to Adikavi Nannaya University, Rajahmahendravaram)

SEMISTER – V , PAPER- V-(RING THEORY & VECTOR CALCULUS)

**Time: 3 Hours** **Maximum Marks: 75**

**SECTION-A**

Answer any **FIVE** questions. Each question carries **FIVE** marks. **5 x 5 = 25 Marks**

1) Prove that every field is an integral domain.

2) If R is a Boolean ring then prove that (i) $a + a = 0 \ \forall\ a \in R$ (ii) $a + b = 0 \Rightarrow a = b$.

3) Prove that Intersection of two sub rings of a ring R is also a sub ring of R.

4) If ***f*** is a homomorphism of a ring R into a ring $R^1$ then prove that Ker ***f*** is an ideal of R.

5) Prove that Curl (grad $\emptyset$) = $\bar{0}$.

6) If $f = xy^2 t + 2x^2yz\, j - 3yz^2 k$ the find div f and Curl f at the point (1, -1 1).

7) If $R(u) = (u - u^2)i + 2u^3 j - 3k$ then find $\int_1^2 R(u)du$.

8) Show that $\int_S (ax\, i + by\, j + cz\, k).N\, dS = 4\pi(a + b + c)$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$

SECTION - B

Answer any FIVE questions at least two from each part. Each question carries Ten marks: 5 X 10 = 50 M

PART – I

9. Prove that a finite integral domain is a field

10. Prove that the characteristic of an integral domain is either a prime or zero.

11. State and prove fundamental theorem of homomorphism of rings.

12. Prove that the ring of integers Z is a Principal ideal ring.

13. If $a = x + y + z$, $b = x^2 + y^2 + z^2$ , $c = xy + yz + zx$ ; then prove that

    [ grad a, grad b, grad c] = 0.

PART-II

14. Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t$, $y = t^2$, $z = t^3$ at the point (1, 1, 1,).

15. Evaluate $\int_S F.N ds$, where $F = z\,\mathbf{i} + x\,\mathbf{j} - 3y^2z\,\mathbf{k}$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between $z = 0$, and $z = 5$.

16. If $F = (2x^2 - 3z)i - 2xy\,j - 4x\,k$ ,then evaluate $\iiint_V \nabla . F dV$ where V is the closed region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 4$.

17. State and Prove Stoke's theorem.

18. Find $\oint_C (x^2 - 2xy)dx + (x^2y + z)dy$ around the boundary C of the region bounded by $y^2 = 8x$ and $x = 2$ by Green's theorem.