MATHEMATICS MODEL PAPER

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SEMISTER – V, PAPER 6 – LINEAR ALGEBRA

Time: 3 Hours **Maximum Marks: 75**

SECTION-A

Answer any **FIVE** questions. Each question carries **FIVE** marks. **5 x 5 = 25 Marks**

1) Let p, q, r be the fixed elements of a field F. Show that the set W of all triads (x, y, z) of elements of F, such that $px + qy + rz = 0$ is a vector subspace of V3 (F).

2) Express the vector $\alpha = (1, -2, 5)$ as a linear combination of the vectors $e_1 = (1, 1, 1)$, $e_2 = (1, 2, 3)$ and $e_3 = (2, -1, 1)$.

3) If α , β , γ are L.I vectors of the vector space V(R) then show that $\alpha + \beta$, $\beta + \gamma$, $\gamma + \alpha$ are also L.I vectors.

4) Describe explicitly the linear transformation $T: R^2 \rightarrow R^2$ such that T (1, 2) = (3, 0), and T (2, 1) = (1,2).

5) Let U(F) and V(F) be two vector spaces and $T : U(F) \to V(F)$ be a linear transformation.

   Prove that the range set R(T) is a sub space of V(F).

6) Solve the system 2x-3y+z=0, x+2y-3z=0, 4x-y-2z=0.

7) State and prove Schwarz inequality.

8) Show that the set $S = \{(1,1,0), (1,-1,1), (-1,1,2)\}$ is an orthogonal set of the inner product space $R^3(R)$.

SECTION - B

Answer any FIVE questions at least two from each part. Each question carries Ten marks: 5 X 10 = 50 M

PART – I

9. Prove that the subspace W to be a subspace of V(F) $\Leftrightarrow a\alpha + b\beta \in W \ \forall a,b \in F$ and $\alpha,\beta \in W$

10. Prove that the four vectors α =(1,0,0), β =(0,1,0), γ =(0,0,1), δ =(1,1,1) in V3(C) form a Linear dependent set, but any three of them are Linear Independent.

11. Let W be a subspace of a finite dimensional vector space V(F), then prove that **dim** $(V/W) =$ **dim** $(V) -$ **dim** (W)

12. Let W_1 and W_2 be two subspaces of a finite dimensional vector space V(F). Then prove that **dim** $(W_1 + W_2) =$ **dim** $(W_1) +$ **dim** $(W_2) -$ **dim** $(W_1 \cap W_2)$.

13. State and prove Rank-Nullity theorem.

PART-II

14. Define a Linear transformation. Show that the mapping T: $R^3 \rightarrow R^2$ is defined by

T (x, y, z) = (x - y, x - z) is a linear transformation.

15. State and prove Cayley- Hamilton theorem.

16. Find the characteristic roots and the corresponding characteristic vectors of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

17. State and prove Bessel's inequality.

18. Applying Gram-Schmidt orthogonalization process to obtain an orthonormal basis of $R^3(R)$ from the basis S = { (1, 1, 0), (-1,1,0), (1, 2, 1,)}.
