

ADIKAVI NANNAYA UNIVERSITY, RAJAMAHENDRAVARAM
III B.A/B.Sc VI Semester MATHEMATICS PAPER-VIII (A,B,D-2)
CLUSTER ELECTIVE - SPECIAL FUNCTIONS

Time:3Hrs

Max.Marks:75

SECTION-A

Answer any 5 questions of the following.

5X5=25

1. Prove that $H_n^{(1)}(x) = 4n(n-1)H_{n-2}$

2. Evaluate $\int_{-\infty}^{\infty} x e^{-x^2} H_n(x) H_m(x) dx$

3. Prove that $L_n^{(1)}(x) = -\sum_{r=0}^{n-1} L_r(x)$

4. Prove that $p_n(0) = 0$ for 'n' odd and $p_n(0) = \frac{(-1)^{\frac{n}{2}} n!}{2^n \left(\frac{n}{2} \right)!}$ for 'n' even

5. Prove that $\int_{-1}^1 P_m(x) P_n(x) dx = 0$ if $m \neq n$

6. Prove that i) $J_{0,1} = -J_1$ ii) $J_2 - J_0 = 2J_{0,1}$

7. Prove that $2J_n^{(1)}(x) = J_{n-1}(x) - J_{n+1}(x)$

8. Evaluate $\int_0^a x^4 \sqrt{a^2 - x^2} dx$

SECTION-B

Answer all the following questions

5X10=50

9. Prove that $e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x)$

10. Prove that $\int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_m(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ \sqrt{\pi} 2^n n! & \text{if } m = n \end{cases}$

11. Prove that $L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$

12. Find $L_0(x)$, $L_1(x)$, $L_2(x)$, $L_3(x)$ and $L_4(x)$

13. Prove that $P_n(x) = \frac{1}{n!} \frac{d^n}{dx^n} (x^2 - 1)^n$

14. Prove that $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$

15. Show that i) $J_{-n}(x) = (-1)^n J_n(x)$ when n is a positive integer

ii) $J_n(-x) = (-1)^n J_n(x)$ when n is a negative integer

16. Prove that $xJ_n'(x) = -nJ_n(x) + xJ_{n-1}(x)$

17. Prove that $\beta_{l,m} = \frac{\Gamma l \Gamma m}{\Gamma l + \Gamma m}$

18. When n is a positive integer, Prove that $\Gamma\left(-n + \frac{1}{2}\right) = \frac{(-1)^n 2^n \sqrt{\pi}}{1.3.5 \dots (2n-1)}$
